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REPLY TO COMMENT

On the shifted $1/N$ expansion method for two-dimensional hydrogenic donor states in an arbitrary magnetic field

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Abstract. Unlike what has been reported by Villalba and Pino (*J. Phys.: Condens. Matter* **8** 8067) the results of the shifted $1/N$ expansion method, obtained by Mustafa (*J. Phys.: Condens. Matter* **5** 1327), for the two-dimensional hydrogenic donor impurity states in an arbitrary magnetic field are accurate and fast converging. The exact analytical results at zero- and high-magnetic-field limits were reproduced in this paper by Mustafa and appeared to be in excellent agreement with those of Martin *et al* (*Phys. Rev.* B **45** 8359).

Recently, Villalba and Pino [1] claimed that 'the results reported by Mustafa [2], who used the shifted $1/N$ expansion method, cannot be valid in the weak-magnetic-field regime'. The purpose of this comment is to show that this claim is irrelevant.

Mustafa [2] has clearly mentioned that the limiting values of the energies at zero- and high-magnetic-field limits are $E_{donor} = -(n_\rho + |m| + 1/2)^{-2}$ and $E_{Landau} = \gamma(2n_\rho + |m| + m + 1)$, respectively (see equations (13) and (14) in [2]). These were the analytical results of the shifted $1/N$ expansion method used in [2], and they are well known exact results. Equations (13) and (14) of [2], being obtained by the leading term E_o (see equation (9) in [2]) where higher-order terms of equation (8) in [2] have vanished identically, lead to the conclusion that the shifted $1/N$ expansion is a fast-converging method, at least at the well known limiting cases of the magnetic field.

Apart from the two points shown in figures 2 and 3 of [2] for $\gamma^{'} = \gamma = 0$, the results, again, are in excellent agreement with the results of [3]. However, with the rechecking of equation (13) in [2], the energies for $2P^-(m = -1, n_\rho = 0)$ and $3D^-(m = -2, n_\rho = 0)$ are in excellent agreement with the results obtained by Martin *et al* [3], without the deviation shown previously in [2].

Furthermore, equations (13) and (14) of [2] are in exact agreement with those of Whittaker and Elliot [4] for the hydrogen impurity case $m_h \to \infty$, infinite-hole-mass limit. This limit has not been considered by Villalba and Pino [1] when they were discussing the Hamiltonian used by [2] and [3]. So it was unfair to underestimate the results of the shifted $1/N$ expansion in the weak-magnetic-field regime. Hence the claim of Villalba and Pino [1] is irrelevant.

The updated results for the $2P^-$ and the $3D^-$ are presented in figures 1 and 2 for comparison with [3].

In conclusion, the shifted $1/N$ expansion method is plausible and effective ([2], [5–7] and references therein). It has advantages over the other approximation methods in its

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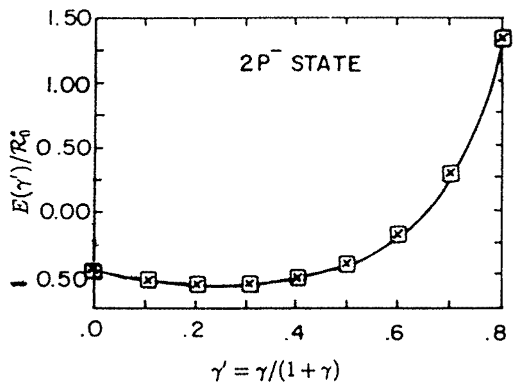

Figure 1. The 2D donor energy versus γ' for the $2P^-$ state; —×—, results of Martin *et al* [3]; —□—, best-fit line of Mustafa's [2] predictions (□).

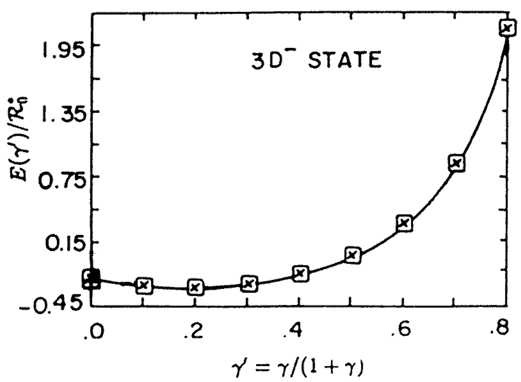

Figure 2. The 2D donor energy versus γ' for the $3D^-$ state; —×—, results of Martin *et al* [3]; —□—, best-fit line of Mustafa's [2] predictions (□).

rapid convergence and tendency to approach perturbation theory results in both the weak- and strong-magnetic-field cases [5]. Quiroga *et al* [7] have shown that this method is an excellent choice to calculate the energy spectrum of hydrogen-like impurity or heavy excitons in an arbitrary magnetic field.

References

[1] Villalba V M and Pino R 1996 *J. Phys.: Condens. Matter* **8** 8067
[2] Mustafa O 1993 *J. Phys.: Condens. Matter* **5** 1327
[3] Martin P, Nunes J and Marquez J 1992 *Phys. Rev.* **B45** 8359
[4] Whittaker D M and Elliot R J 1988 *Solid State Commun.* **68** 1
[5] Mustafa O and Chhajlany S C 1994 *Phys. Rev.* **A50** 2926
[6] Mustafa O and Sever R 1991 *Phys. Rev.* A **44** 4142
Mustafa O and Sever R 1993 *Hadronic J.* **16** 57
[7] Quiroga L, Camacho A and Gonzales A 1995 *J. Phys.: Condens. Matter* **7** 7517